Minimum Number of Distinct Eigenvalues of Graphs

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Abstract

For a graph G on n vertices, let $\mathcal{S}(G)$ be the set of all $n \times n$ real symmetric matrices such that their nonzero off-diagonal entries represent the weights of the edges of G. The inverse eigenvalue problem for a graph G (IEP-G) asks to determine all possible spectra of matrices in $\mathcal{S}(G)$.

A list of positive integers $\mathbf{m} = (m_1, m_2, \ldots, m_k)$ is realized as an ordered multiplicity list for the graph G if there is a matrix in $\mathcal{S}(G)$ with k distinct eigenvalues such that the ith largest eigenvalue has the multiplicity m_i, for $i = 1, 2, \ldots, k$.

One of the relaxations of the IEP-G is to determine the minimum length among all realizable multiplicity lists of a graph. This parameter is denoted by $q(G)$ and it is called the minimum number of distinct eigenvalues of G.

In this presentation, we will review interesting advances and techniques from a number of recent developments regarding $q(G)$.

References

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