Determinantal Representations in Theory and Applications

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Abstract

A linear determinantal representation expresses a multivariate polynomial as the determinant of a square matrix whose entries are linear forms. The study of determinantal representations dates back to 19th century classical algebraic geometry and has since found applications in partial differential equations, operator theory, convex optimization, and complexity theory. I will survey some of the classical and recent theory of determinantal representations with a focus on applications in linear algebra and matrix theory, including numerical ranges and the principal minor map.

References

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