Eigenvalue nonlinearities and eigenvector nonlinearities

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Abstract

The following two generalizations of the standard eigenvalue problem have received considerable attention in the numerical linear algebra community: The eigenvalue nonlinear eigenvalue problem $A(\lambda)x = 0$, where $A : \mathbb{C} \to \mathbb{C}^{n\times n}$ is typically a holomorphic or meromorphic function of the scalar λ, and the eigenvector nonlinear eigenvalue problem $A(x)x = \lambda x$, where either $A : \mathbb{C}^n \to \mathbb{C}^{n\times n}$ is assumed to be homogeneous, $A(\alpha x) = A(x)$, or we explicitly require a normalization condition, e.g., $x^H x = 1$. We summarize how these problems arise in applications, for example delay differential equations, acoustics, quantum physics and data science. Application-driven numerical developments are presented, as well as a review of general numerical linear algebra, and theoretical approaches for both types of problems in the context of specific structures.

References

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